# Deflection of light by the screw dislocation in space-time

*Miroslav Pardy*
*Department of Theoretical Physics and Astrophysics*
*Masaryk University*
*Kotlářská 2, 611 37 Brno, Czech Republic*
e-mail:pamir@physics.muni.cz

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**Abstract**

We derive the light deflection caused by the screw dislocation in space-time. The derivation is based on the idea that space-time is a medium which can be deformed by gravity and that the deformation of space-time is equivalent to the existence of gravity.

# 1 Introduction

There is a possibility that during the big bang, supernova explosion, gravitational collaps, collisions of the high-energy elementary particles and so on, the dislocations in space-time are created. In this article we derive the deflection of light caused by the screw dislocation in space-time.

In order to derive such deflection of light, it is necessary to explain the origin of metric in the Einstein theory of gravity.

Einstein gives no explanation of the origin of the metrics, or, metrical tensor. He only introduces the Riemann geometry as the basis for the general relativity [1]. He "derived" the nonlinear equations for the metrical tensor [2] and never explained what is microscopical origin of the metric of space-time. Einstein supposed that it is adequate that the metric follows from differential equations as their solution. However, the metric has an microscopical origin similarly to situation where the phenomenological thermodynamics has also the microscopical and statistical origin.

The first question we ask, is, what is the microscopical origin of the metric of space-time. We postulate that the origin of metric is the specific deformation of space-time continuum. We take the idea from the mechanics of continuum and we apply it to the space-time medium. The similar approach can be found in the Tartaglia article and author eprint, [3], where space-time is considered as a deformable medium.

The mathematical description of the three dimensional deformation is given for instance in [4]. The fundamental quantity is the tensor of deformation expressed by the relative displacements $u^i$ as follows:

$$u_{ik} = \left(\frac{\partial u_i}{\partial x^k} + \frac{\partial u_k}{\partial x^i} + \frac{\partial u^l}{\partial x^i}\frac{\partial u_l}{\partial x^k}\right); \quad i,k = 1,2,3. \tag{1}$$

The last definition can be generalized to the four dimensional situation by the following relation:

$$u_{\mu\nu} = \left(\frac{\partial u_\mu}{\partial x^\nu} + \frac{\partial u_\nu}{\partial x^\mu} + \frac{\partial u_\alpha}{\partial x^\mu}\frac{\partial u^\alpha}{\partial x^\nu}\right); \quad \mu,\nu = 0,1,2,3, \tag{2}$$

with $x^0 = ct, x^1 = x, x^2 = y, x^3 = z$.

In order to establish the connection between metric $g_{\mu\nu}$ and deformation expressed by the tensor of deformation, we write for the metrical tensor $g_{\mu\nu}$ in squared space-time element

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu, \tag{3}$$

the following relation

$$g_{\mu\nu} = (\eta_{\mu\nu} + u_{\mu\nu}), \tag{4}$$

where

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{5}$$

Instead of work with the metrical tensor $g_{\mu\nu}$, we can work with the tensor of deformation $u_{\mu\nu}$ and we can consider the general theory of relativity as the four-dimensional theory of some real deformable medium as a partner of the metrical theory. First, let us test the deformation approach to the space-time in case of the nonrelativistic limit.

## 2 The nonrelativistic test

The Lagrange function of a point particle with mass $m$ moving in a potential $\varphi$ is given by the following formula [5]:

$$L = -mc^2 + \frac{mv^2}{2} - m\varphi. \tag{6}$$

Then, for a corresponding action we have

$$S = \int L dt = -mc \int dt \left( c - \frac{v^2}{2c} + \frac{\varphi}{c} \right), \tag{7}$$

which ought to be compared with $S = -mc \int ds$. Then,

$$ds = \left( c - \frac{v^2}{2c} + \frac{\varphi}{c} \right) dt. \tag{8}$$

With $d\mathbf{x} = \mathbf{v}dt$ and neglecting higher derivative terms, we have

$$ds^2 = (c^2 + 2\varphi)dt^2 - d\mathbf{x}^2 = \left( 1 + \frac{2\varphi}{c^2} \right) c^2 dt^2 - d\mathbf{x}^2. \tag{9}$$

The metric determined by this $ds^2$ can be be obviously related to the $u_\alpha$ as follows:

$$g_{00} = 1 + 2\partial_0 u_0 + \partial_0 u^\alpha \partial_0 u_\alpha = 1 + \frac{2\varphi}{c^2}. \tag{10}$$

We can suppose that the time shift caused by the potential is small and therefore we can neglect the nonlinear term in the last equation. Then we have

$$g_{00} = 1 + 2\partial_0 u_0 = 1 + \frac{2\varphi}{c^2}. \tag{11}$$

The elementary consequence of the last equation is

$$\partial_0 u_0 = \frac{\partial u_0}{\partial (ct)} = \frac{\varphi}{c^2}, \tag{12}$$

or,

$$u_0 = \frac{\varphi}{c} t + const. \tag{13}$$

Using $u_0 = g_{00} u^0$, or, $u^0 = g_{00}^{-1} u_0 = \frac{\varphi}{c} t$, we get with $const. = 0$ and

$$u^0 = ct' - ct, \tag{14}$$

the following result

$$t'(\varphi) = t(0) \left( 1 + \frac{\varphi}{c^2} \right), \tag{15}$$

which is the Einstein formula relating time in the zero gravitational field to time in the gravitational potential $\varphi$. The time interval $t(0)$ measured remotely is so called the coordinate time and $t(\varphi)$ is local proper time. The remote observe measures time intervals to be delated and light to be red shifted. The shift of light frequency corresponding to the gravitational potential is, as follows [5].

$$\omega = \omega_0 \left(1 + \frac{\varphi}{c^2}\right). \tag{16}$$

The precise measurement of the gravitational spectral shift was made by Pound and Rebka in 1960. They predicted spectral shift $\Delta\nu/\nu = 2.46 \times 10^{-15}$ [1]. The situation with red shift is in fact closed problem and no additional measurement is necessary.

While we have seen that the red shift follows from our approach immediately, without application of the Einstein equations, it is evident that the metrics determined by the Einstein equations can be expressed by the tensor of deformation. And vice versa, to the every tensor of deformation the metrical tensor corresponds.

## 3 The deflection of light by the screw dislocation

According to [4] the screw deformation is defined by the tensor of deformation which is in the cylindrical coordinates as

$$u_{z\varphi} = \frac{b}{4\pi r}, \tag{17}$$

where $b$ is the $z$-component of the Burgers vector. The Burgers vector of the screw islocation has components $b_x = b_y = 0, b_z = b$.

The postulation of the space-time as a medium enables to transfer the notions of the theory of elasticity into the relativistic theory of space-time and gravity. The considered transfer is of course the heuristical operation, nevertheless the consequences are interesting. To our knowledge, the problem, which we solve is new.

We know that the metric of the empty space-time is defined by the coefficients in the relation:

$$ds^2 = c^2dt^2 - dr^2 - r^2d\varphi^2 - dz^2. \tag{18}$$

If the screw deformation is present in space-time, then the generalized metric is of the form:

$$ds^2 = c^2dt^2 - dr^2 - r^2d\varphi^2 - 2u_{z\varphi}dzd\varphi - dz^2, \tag{19}$$

or,

$$ds^2 = c^2dt^2 - dr^2 - r^2d\varphi^2 - \frac{2b}{4\pi r}dzd\varphi - dz^2. \tag{20}$$

The motion of light in the Riemann space-time is described by the equation $ds = 0$. It means, that from the last equation the following differential equation for photon follows:

$$0 = c^2 - \dot{r}^2 - r^2\dot{\varphi}^2 - \frac{b}{2\pi r}\dot{z}\dot{\varphi} - \dot{z}^2. \tag{21}$$

Every parametric equations which obeys the last equation are equation of motion of photon in the space-time with the screw dislocation. Let us suppose that the motion of light is in the direction of the z-axis. Or, we write

$$r = a; \quad \dot{z} = v. \tag{22}$$

Then, we get equation of $\varphi$:

$$2\pi a^3\dot{\varphi}^2 + bv\dot{\varphi} = 2\pi a(c^2 - v^2). \tag{23}$$

We suppose that the solution of the last equation is of the form

$$\varphi = At. \tag{24}$$

Then, we get for the constant $A$ the quadratic equation

$$2\pi a^3 A^2 + bvA + 2\pi a(v^2 - c^2) = 0 \tag{25}$$

with the solution

$$A_{1/2} = \frac{-bv \pm \sqrt{b^2v^2 - 16\pi^2a^4(v^2 - c^2)}}{4\pi a^3}. \tag{26}$$

Using approximation $v \approx c$, we get that first root is approximately zero and for the second root we get:

$$A \approx \frac{-bc}{2\pi a^3}, \tag{27}$$

which gives the function $\varphi$ in the form:

$$\varphi \approx \frac{-bc}{2\pi a^3}t. \tag{28}$$

Then, if $z_2 - z_1 = l$ is a distance between two points on the straight line parallel with the axis of screw dislocation then, $\Delta t = l/c$, $c$ being the velocity of light. For the deflection angle $\Delta\varphi$, we get:

$$\Delta\varphi \approx \frac{-bl}{2\pi a^3}. \tag{29}$$

So, we can say, that if we define the screw dislocation by the metric of eq. (20), then, the deflection angle of light caused by such dislocation is given by eq. (29).

## 4 Discussion

We have defined gravitation as a deformation of a medium called space-time. We have used equation which relates Riemann metrical tensor to the tensor of deformation of the space-time medium and applied it to the gravitating system, which we call screw dislocation in space-time. The term screw dislocation was used as an analogue with the situation in the continuous mechanics. We derived the angle of deflection of light passing along the screw dislocation axis at the distance $a$ from it on the assumption that trajectory length was $l$. This problem was not considered for instance in the Will monography [6].The screw dislocation was still not observed in space-time and it is not clear what role play the dislocations in the development of universe after big bang. Our method can be applied to the other types of dislocations in space-time and there is no problem to solve the problem in general. We have used here the specific situation

because of its simplicity. We have seen that the problem of dislocation in space-time is interesting and it means there is some scientific value of this problem. Sooner or later the physics will give answer to the question what is the the role of dislocations in forming of universe.

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